M317 ec 2 Assignment #2

1. Under what conditions is $\sup(A)$ not an accumulation point for A?

   If $\alpha = \sup(A)$ is an isolated point of A (in which case, it must belong to A)
   For example $\alpha = 3 = \sup\{1,2,3\}$ is not an accumulation point of the set.

2. Give an example of a set with the following properties or explain why no such set is possible:

   a) an infinite set with no accumulation points
      $\mathbb{N}$ the natural numbers has no acc pts

   b) a bounded set with no accumulation points
      $A = \{1,2,3\}$ is bounded and has no acc pts

   c) an interval (a,b) containing only irrational numbers
      not possible, between any two real numbers $a < b,$ there is an irrational

   d) a set $A \subset R$ that contains its $\sup$ but not its $\inf$
      $(1,2) \cup \{10\}$ contains its sup, 10, but not its $\inf,$ 1

   e) a finite set that does not contain its $\sup$

      not possible, the $\sup$ of a finite set is just the largest number in the set.

3. Show that every irrational number is an accumulation point of R

   Let $x =$ irrational number and let $\varepsilon > 0$ denote an arbitrary positive number. Then by the Archimedes principle, there exists a rational number, r, between $x - \varepsilon$ and $x + \varepsilon$. Since r is rational, it does not equal x so $\forall \varepsilon > 0,\ \exists r,\ r \in \mathring{N}_\varepsilon(x)$.